ASSIGNMENT PROBLEM

The assignment problem refers to another special class of linear programming problem where the objective is to assign a number of resources to an equal number of activities on a one-to-one basis so as to minimize total costs of performing the tasks at hand or maximize total profit.

Mathematical Model of Assignment Problem:

Given ‘n’ facilities (resources) and ‘n’ jobs (activities) and effectiveness (in terms of cost, time, profit) of each facility for each job, the problem lies in assigning each resource to one and only one activity so that the given measure of effectiveness is optimized.

The cost matrix is same as that of transportation cost matrix except that capacity at each of the resources and the requirements at each of the destinations is taken to be unity because assignments are made on a one to one basis.

Let Xij denote the assignment of 'ith' facility to 'jth' activity such that

Xij = [1, if 'ith' facility is assigned to 'jth' activity]
[0, otherwise]

Balanced Assignment Problem: An assignment problem is said to be balanced if the number of persons is equal to the number of jobs.

Unbalanced Assignment Problem: An assignment problem is said to be unbalanced if the number of persons is not equal to the number of jobs. To make unbalanced assignment problem, a balanced one, a dummy person or a dummy job is introduced with zero cost or time.

Assignment Algorithm (The Hungarian Method): The computational procedure for solving assignment problem can be summarized in the following steps:

I:- Develop the Opportunity Cost Table:

(a) **Row Reduction:** Subtract the minimum entry of each row from all the entries of the respective row in the cost matrix.

(b) **Column Reduction:** After completion of row reduction, subtract the minimum entry of each column from all the entries of the respective column.

II: Make assignments in the opportunity cost matrix:

(a) Starting with Ist row of the matrix received in first step, examine the rows successively one by one until a row containing exactly one zero is found. Now cross all the zeros in the column in which the assignment is made. This procedure should be adopted for each row assignment.

(b) When the set of rows has been completely examined, an identical procedure is applied successively to columns. Starting with column 1, examine all columns until a column containing exactly one zero is found.

(c) Repeat the operations until one of the following situation arises:

(III) **<u>Optimality Criterion:</u>**

(a) If all the zeros in rows/columns are either marked cross and there is exactly one assignment in each row and in each column. In such a case optimal assignment policy for the given problem is obtained.

(b) There may be some row (or columns) without assignment i.e. the total number of marked zeros is less than the order of the matrix. In such a case, go to the next step.

(IV) Revise the opportunity cost matrix.

(V) Develop the new revised opportunity cost matrix.

Q1: The distance between the cities are given in the following distance matrix:

	I	II	III	IV
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	14	10	12	11

Q2: Five machines are available to do 5 different jobs. Find the assignment of machines to jobs that will minimize the total time taken:

	I	II	III	IV	V
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Q3: The number of man-hours needed to complete a job for each job-man combination are given below:

	A	B	C	D	E
I	12	8	7	15	4
II	7	9	17	14	10
III	9	6	12	6	7
IV	7	6	14	6	10
V	9	6	12	10	6

Q4: The following table shows the annual sales that can be generated by each salesman in each territory. Find the optimum assignment.

	T1	T2	T3	T4	T5
S1	32	38	40	28	40
S2	40	24	28	21	36
S3	41	27	33	30	37
S4	22	38	41	36	36
S5	29	33	40	35	39

(2) Unbalanced Assignment Problem:

When the cost matrix of an assignment problem is not a square matrix, i.e. number of rows are not equal to number of columns, then the assignment problem is called an unbalanced transportation problem. In such cases, dummy rows/columns are added in the matrix with zero costs. Then, the usual Hungarian method is used to solve the resulting balanced problem.

Q5: solve the following unbalanced assignment problem:

	I	II	III	IV	V
A	6	2	5	3	6
B	2	5	8	7	7
C	7	8	6	9	8
D	6	2	3	4	5
E	9	3	8	9	7
F	4	7	4	6	8

Q6: A company has 6 machines and 5 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table:

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10

Q7: Solve the adjoining unbalanced assignment problem of minimizing total time for doing all the jobs:

	1	2	3	4	5
1	6	2	5	3	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

Special Variations in the assignment Problem:

(1) **Maximisation case in assignment problem:** Sometimes, the assignment problem may relate to the maximisation of profit by assigning people to tasks or jobs to machines. Such problems are first reduced to minimization problem before applying the hungarian method. This is achieved by subtracting from the highest element in the assignment matrix, all other elements of the matrix.

Q: A captain of a cricket team has to allot five middle order batting positions to five batsmen. The average runs scored by each batsmen at these positions are given in the table:

	I	II	III	IV	V
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	48	60	59	55	53

Introduction
of
Optimization Techniques

- **According to Churchman, Arnoff,** “Operations Research is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solutions to the problem.”

- Operations Research consists of techniques which were designed for solving assorted problems relating to transportation, assignment of tasks to persons or salesmen in various cities, allocation of resources for minimizing loss or maximising profits and reducing wastages.

Methodology of Operations Research:

(a) **Formulating the Problem:** The primary phase of OR is to develop a clear and concise statement of the problem. The problem must be formulated in the form of an appropriate model. For this, the following components should be taken into consideration:

(i) What are the objectives?

(ii) What are the uncontrollable variables that may effect the possible solutions?

(iii) What are the restrictions on the variables?

(b) **Constructing a Mathematical Model:** The next step is the construction of a suitable mathematical model. Model construction means translating the problem definition into mathematical relationships. A model should include the following three important things:

(i) Decision variables.

(ii) Constraints or restrictions.

(iii) Objective function.

(c) **Model Solution:** This phase involves the computation of those values of decision variables that optimise the objective function. An optimal solution is one which maximise or minimise the performance of any measure in a model subject to the conditions and constraints represented in the model.

(d) <u>Model Testing:</u> It involves checking as if the model can reliably predict the actual system's performance. It also includes comparing the performance of the model with past data available for the actual system. A model must be applicable for a reasonable time period and should be updated from time to time, taking into consideration the past, present and future aspects of the problem.

(e) <u>Establishing control over the solution:</u> After testing the model, the next step is to establish control over the solution, by proper feedback of the information on variables. In other words, if one or more of the controlled variables change, the model may be modified accordingly.

(f) <u>Implementation of the final results:</u> The optimal solution obtained from the model should be applied in practice to improve the performance of the system and the validity of the solution be verified under changing conditions.

Allocation Models: Allocation models are used to allocate available resources to activities in such a way to minimize or maximize subject to given conditions.

(a) **Waiting Line (or queuing models):** It deals with the situation in which queue is formed. These models have been developed to establish a trade-off between cost of providing service and the waiting time of a customer in the queuing system. **It involves the following:**

(i) How much average time will be spent by the customer in a queue?

(ii) What will be an average length of a queue?

(iii) Average time spent by a customer in the queue.

(b) Inventory Models: Inventory control model helps in minimizing the sum of three conflicting inventory costs; the cost of holding or carrying extra inventory, the cost of shortage or delay in the delivery of items when it is needed and cost of ordering.

(c) Network Models: These models are used to plan, schedule and monitor large projects having complexities and inter-dependence of activities. PERT and CPM techniques helps in identifying trouble spots in project through the identification of the critical path.

(d) Sequencing Models: These models involve the selection of such a sequence of performing a series of jobs to be done on service facilities that optimize the performance of the system.

(e) Replacement Models: These models are related to finding an optimal replacement policy to replace the machines, plant, automobiles, etc.

(f) Assignment Models: It involves the assignment of a number of jobs to the same number of men.

SEQUENCIN G PROBLEM

- Sequencing problem arises when a choice has to be made as to the order in which a number of tasks can be performed.

- Selection of an appropriate order for a series of jobs to be done on a finite number of service facilities, in some pre-assigned order, is called sequencing.

- Sequencing is the determination of the order in which tasks are performed and scheduling is the determination of times for each task.

- Sequencing problem is the problem in which it is necessary to determine the order or sequence of jobs in which they should be performed so as to minimize the total costs.

- **<u>Different types of Sequencing Problems</u>:**

(a) Processing ‘n’ jobs through two machines.

(b) Processing ‘n’ jobs through three machines.

(c) Processing ‘n’ jobs through ‘k’ machines.

(d) Processing 2 jobs through ‘k’ machines.

Notations:

(a) **tij** = time required for processing *i*th job on the *j*th machine.

(b) **T** = Total elapsed time for processing all the jobs, including idle time.

(c) **Xij** = Idle time on machine *j* from the end of job (*i*- 1) to the start of job *i*.

Terminology:

(a) Number of Machines: It represents the number of service facilities through which a job must pass before it is assumed to be completed.

(b) Processing time: It indicates the time required by a job on at each machine.

(c) Processing Order: It is the sequence in which given machines are required to

(d) Total elapsed time: It is the time interval between starting the first job and completing the last job including the idle time.

(e) Idle time on a machine: It is the time for which a machine does not have a job to process.

(f) No passing rule: It is the rule of maintaining the order in which jobs are to be processed on given machines.

Processing N Jobs through Two machines:

- Here the objective is to determine the sequence S in which the 'n' jobs may be performed so that T (total elapsed time) is minimized.

- Total elapsed time is determined by the point of time at which the first job goes on machine A and the point of time on which the last job comes off machine B.

- As passing is not allowed, thus machine A will remain busy in processing all the n jobs one by one while machine B may remain idle after completion of one job and before starting of another job.

- At any moment of time, machine B may be idle. Let **x2j** be the time for which machine **M2** remains idle after finishing (j-1)th job and before starting processing the *i*th job.

Johnson's Rule:

- It is a method of sequencing which was propounded by American Mathematician Johnson in 1954.
- It provides the various jobs along with their processing time on machine A and Machine B.
- Examine the columns for processing times on machine A and B and find the smallest processing time in each column.
- If the minimum is for machine A, then place the corresponding job in the first available position in the sequence and if it is for machine B, then place the corresponding job in the last available position in the sequence.

- **If there is a tie in selecting the minimum of all the processing times, then there may be three situations:**

(a) Minimum among all processing times is same for A and B machines, then process job on Machine A first and Machine B last.

(b) If the tie for minimum occurs among processing times on machine A only, then select arbitrarily the job to process first.

(c) If the tie for the minimum occurs among processing times on machine B only, then select arbitrarily the job to process last.

- Cross off the jobs already assigned and repeat steps II through IV.

- Calculate idle time for machine A and B

- Calculate idle time for machine A and B:

(a) Idle time for Machine A = Total Elapsed time – time when the last job in a sequence finishes on A.

(b) Idle time for Machine B = Time at which the first job in a sequence starts on A + (time when the job in a sequence starts on B – time when the last job in a sequence finishes on B).

- The total elapsed time to process all jobs through two machines is given by:

Total Elapsed Time= Time when the last job in a sequence finishes on Machine B.

Q1: There are five jobs, each of which must go through the two machines A and B in the order AB. Processing time are given in the table below:

Determine a sequence for the given jobs that will minimise the total elapsed time.

Job No.	J1	J2	J3	J4	J5
Processing time A	5	1	9	3	10
Processing time B	2	6	7	8	4

Q2: There are six jobs, each of which must go through the two machines A and B in the order AB. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A and Machine B.

Job No.	J1	J2	J3	J4	J5	J6
Processing time A	1	3	8	5	6	3
Processing time B	5	6	3	2	2	10

Q3 : There are seven jobs, each of which must go through the two machines A and B in the order AB. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A and Machine B.

Job No.	J1	J2	J3	J4	J5	J6	J7
Processing time A	6	24	30	12	20	22	18
Processing time B	16	20	20	12	24	2	6

Processing “N” jobs through 3 Machines:

- Here the case is similar to that of processing n jobs through 2 machines except that three machines are involved.
- Each job requires the same sequence of operations and no passing is allowed and if either or both the following condition/s are satisfied:

(a) The minimum time on machine A is greater than or equal to the maximum time on machine B.

(b) The minimum time on machine C is greater than or equal to the maximum time on machine B.

i.e. if either minimum of A $\geq$ maximum of B, or

minimum of C $\geq$ maximum of B, then the following can be applied:

- Replace the problem with an equivalent problem involving n jobs and two machines. Denote the fictitious machines by G and H and define the processing time **Gi** and **Hi** by:

Gi = Ai + Bi , **Hi = Bi + Ci**

- Now, workout the new problem, two machines and n jobs, with the prescribed order GH, by the same method of processing "n" jobs through 2 machines.

Q1 : There are five jobs, each of which must go through the three machines A, B and C in the order ABC. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A, B and C.

Job No.	J1	J2	J3	J4	J5
Processing time A	4	9	8	6	5
Processing time B	5	6	2	3	4
Processing time C	8	10	6	7	11

Q2 : There are seven jobs, each of which must go through the three machines A, B and C in the order ABC. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A, B and C.

Job No.	J1	J2	J3	J4	J5	J6	J7
Processing time A	30	80	70	40	90	80	70
Processing time B	40	30	20	50	10	40	30
Processing time C	60	70	50	110	50	60	120

<u>Processing "N" jobs through 3 Machines (when initial conditions are not satisfied)</u>

If the initial conditions are not satisfied, we will change the conditions as follows:

(a) The minimum time on machine A is greater than or equal to the maximum time on machine C.

(a) The minimum time on machine B is greater than or equal to the maximum time on machine C.

i.e. if either minimum of A ≥ maximum of C, or

minimum of B ≥ maximum of C, then the following can be applied:

- Replace the problem with an equivalent problem involving n jobs and two machines. Denote the fictitious machines by G and H and define the processing time **Gi** and **Hi** by:

$$\mathbf{Gi} = \mathbf{Ai} + \mathbf{Ci}, \qquad \mathbf{Hi} = \mathbf{Bi} + \mathbf{Ci}$$

- Now, workout the new problem, two machines and n jobs, with the prescribed order GH, by the same method of processing "n" jobs through 2 machines.

- Now, the sequence of machine will be changed to ACB.

Q23: There are seven jobs, each of which must go through the three machines A, B and C in the order ABC. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A, B and C.

Job No.	J1	J2	J3	J4	J5	J6
Processing time A	12	8	7	11	10	5
Processing time B	7	10	9	6	10	4
Processing time C	3	4	2	5	1.5	4

LINEAR PROGRAMMING

Meaning:

Linear programming is a technique for choosing the best alternative from a set of feasible alternatives, in situation in which the objective function as well as the constraints can be expressed as linear mathematical functions.

Applications of Linear Programming Problems:

(a) Optimal Product Line problems

(b) Product mix problem

(c) Diet planning problems

(d) Transportation problems.

Basic Requirements of a Linear Programming Problem:

(a) **Objective Function:** A function of certain variables which is to be optimized subject to given conditions on the variables of the function is called objective function of the problem. If it is profit then it will be maximization type and if it is cost then it will be minimization type.

(b) **Non-negative restrictions:** Non-negative restriction indicates that all decision variables must take on values equal to or greater than zero.

(c) **Constraints:** The constraints indicate limitations on the resources which are to be allocated among various decision variables. These resources may be production capacity, man power, time or machinery, etc.

Assumptions of Linear Programming:

(a) **Linearity:** The term linearity means straight line among the relevant variables. It is the amount of resource required for a given activity level is directly proportional to the level of that activity.

(b) **Divisibility:** It means that fractional values of the decision variables are permitted.

(c) **Certainty:** The various parameters, i.e. the objective function coefficients, the coefficients of equations and the constraint are known with certainty.

(d) **Non-Negativity:** It means that the decision variables are permitted to have only the values which are greater than or equal to zero.

(e) **Additivity:** Total output for a given combination of activity levels is the algebric sum of the output for each individual process.

(f) **Continuity:** It means that the decision variables are continuous.

Limitations of Linear Programming:

(a) The basic assumption that objective function and constraints are linear may not hold good in many practical situations.

(b) This technique does not consider the problems related to uncertainty.

(c) When the number of variables involved in the problem are quite large, then the procedure become complex and is difficult to handle manually.

(d) Parameters appearing in L.P. model are assumed to be constant but in reality they are frequently neither known or constant.

Q: S company is producing two products A and B. The processing times are 3 and 4 hours per unit for A on operation one and two and 4 hours and 5 hours per unit for B on operation one and two respectively. The available time is 18 hours and 21 hours for operation one and two respectively. The product A can be sold for Rs 3 profit per unit and B of Rs 8 per unit. Solve for maximum profit program. Only formulate the problem.

Q: A dietician mixes two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of Vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C . One kg of food X costs Rs 5 whereas one kg of food Y costs Rs. 7. Determine the minimum cost of such a mixture. Formulate the above as a linear programming problem.

Q: An animal feed company must produce 200 kgs of a mixture consisting of ingredients A and B daily. A costs Rs. 3 per kg and B costs Rs. 8 per kg. No more than 80 kgs of A can be used and at least 60 kgs of B must be used. Find how much of each ingredient should be used if the company wants to minimize cost?

Q: A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are priced Rs. 100 and Rs. 120 per unit respectively, how should the producer use his resources to maximise the total revenue. Use LPP.

Q: A manufacturer has three machines I, II and III installed in his factory. Machine I and II are capable of being operated for at the most 12 hours, whereas machine III must be operated at least for 5 hours a day. He produces only two items, each requiring the use of three machines. One unit of A requires 1 hour on machine I, 2 hours on machine II and 1 hour on machine III. Whereas, one unit of B requires 2 hours on machine I, 1 hour on machine II and 5/4 hours on machine III. He makes a profit of Rs. 60 on item A and Rs. 40 on item B. Assuming that he can sell all that he produces, how many of each item should be produced so as to maximise his profit? Formulate the above problem as LPP.

Q: A company manufactures two types of dolls A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours for assembling in a day. The profit is Rs. 50 each on type A and Rs. 60 each on type B. How many dolls of each type should the company manufacture in a day to maximise the profit? Formulate L.P.P

Q: A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are prices at Rs. 100 and Rs 120 per unit respectively, how should the producer use his resources to maximise the total revenue? Formulate LPP.

Q: A firm manufactures two types of products A and B and sells them at a profit of Rs. 5 per unit of type A and Rs. 3 per unit of type B. Each product is processed on two machines M1 and M2. One unit of type A requires one minute of processing time on M1 and two minutes of processing time on M2; whereas one unit of type B requires one minute of processing time on M1 and one minute on M2. Machines M1 and M2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm should produce in a day in order to maximise the profit. Formulate LPP.

Methods of Linear Programming:

(a) Graphic Method.

(b) Simplex Method.

GRAPHIC METHOD:

There are two techniques of solving an LPP by graphical method:

(c) Corner Point Method

(d) Iso-profit or Iso-cost method.

(A) Corner Point Method:

The maximum/minimum value of a linear objective function over a convex polygon bounded by a number of lines, occurs at some vertex or the other. Following is the procedure to solve a LPP graphically:

(i) Consider each constraint as an equation.

(ii) Plot each equation on graph, as each one will geometrically represent a straight line.

(iii)The common region obtained satisfying all the constraints and the non-negative restrictions is called the feasible region.

(iv) Determine the co-ordinates of the corner points of the convex polygon.

(v) Find the values of the objective function at each of the extreme points. The point at which the value of the objective function is maximum or minimum is the optimal solution of the given LPP.

Special Cases in Linear Programming:

(a) **Multiple Optimal Solution:** Sometimes a LPP may yield more than one optimal solution. It happens only when the objective function is parallel to one of the constraint.

(b) **Unbounded Solution:** An unbounded solution is a solution whose objective function is infinite. The objective function can also be increased infinitely.

(c) **Infeasible solution:** When it is not possible to find a feasible region that satisfies all the constraints then LPP is said to have an infeasible solution.

INVENTOR Y CONTROL

- Inventories refers to any kind of resource that has economic value and is maintained to fulfill the present and future needs of an organization.

- **Inventories may be classified as:**

(a) Physical resources such as raw materials, semi-finished & finished goods

(b) Human resources such as unused labour.

(c) Financial resources such as working capital.

Inventory Control:

•Inventory control is the function of directing the movement of goods through the entire manufacturing cycle from the requisitioning of raw materials to the inventory of finished goods in orderly manner to meet the objectives of maximum customer service with minimum investment and low-cost plant operation.

•To determine what to order? When to order? How much to order? And how much to carry in stock.

•To minimize shortage, holding and replacement costs of inventories leading to maximum efficiency in production and distribution.

•To maintain waste, surplus, scrap and obsolete items at the minimum.

Functional Classification of Inventory:

Inventory can be defined as the stock of goods or any other resources that are stored at any given period for future production or for meeting future demand.

(a) Direct Inventories: It includes those items which play a direct role in the manufacture and become an integral part of finished goods. It includes raw materials, semi-finished goods and finished goods.

(b) Indirect Inventories: It includes those items which are necessary for manufacturing but do not become component of the finished goods. It includes oil, office materials, maintenance materials etc. It may also includes the following:

(i) Pipeline or Transit Inventories: This includes maintenance of optimal inventory level for shipment to distribution centres and customers from production centres. It is essential to keep extra stock of inventory items at various work places to meet the demand while the supply is in transit. The amount of pipeline inventory depend on the time required for shipment and the nature of demand.

(ii) Buffer Inventories or Safety Stock: It is the specific level of extra stock of inventory that is maintained for protection against uncertainties of demand and the lead time necessary for delivery of goods. It is determined by trade-off between protection against demand and supply uncertainties and the level of investment in additional stock.

(iii) <u>Lot Size or Cycle Inventories</u>: These inventories are held due to the fact that orders are placed in lots rather than purchasing the exact amount of inventory which may be needed at a point of time. It is more economical to order inventories in lots to achieve reduced ordering cost or to obtain quantity discounts.

(iv) <u>Decoupling Inventories</u>: These inventories serve the function of decoupling operations in a production system. These permit the various production activities to operate more independently, they do not have to rely completely on the schedule of output of prior activities in the production process.

(v) **Seasonal Inventories**: These inventories are required the seasonal fluctuations in demand. These inventories are used to smooth out the level of production so that workers do not have to be frequently hired to meet such demand fluctuations.

(vi) **Fluctuation Inventories**: These inventories are required because sales and production time of a product cannot always be predicted accurately.

Inventory Decisions:

There are three types of decisions to be made in managing inventories:

(a) How much is to order for each replenishment?

(b) When it is necessary to place an order to replenish inventory?

(c) How much safety stock should be kept?

Decisions regarding the size and timing of replenishment orders are influenced by:

(a)Forecast of demand for an item.

(b)Its replenishment lead time.

(c)Inventory related cost and management policies.

Objectives of Inventory Control:

(a) To reduce the financial investment in inventories.

(b) To ensure that the value of materials consumed is minimum: It involves efficient purchasing, storage, consumption and accounting for materials from the time orders are placed with the suppliers till the materials have been utilized in production.

(c) To maintain timely records of inventories of all items and to maintain the stock within the desired limits.

(d)To ensure timely action for replenishment.

(e) To avoid losses from inventory obsolescence.

(f) To improve customer service: Carrying of raw materials and finished products reduces lead time for deliveries.

(g) To allow flexibility in production scheduling: It relieves the pressure on production system and gives them scheduling flexibility that can reduce unit production costs.

(h) To reduce surplus stock.

(i) To protect the inventories from pilferage, theft, waste, loss and damage.

(j) Helps in tiding over the demand fluctuations.

Benefits of Inventory Management:

(a)Ensures an adequate supply of items to the customers and avoid shortages.

(b)Use of available capital in a most effective way and avoids unnecessary on high inventories.

(c)Reduces the risk of loss due to the changes in price of items.

(d)Takes advantages of quantity discounts on bulk purchases.

(e)Services as a buffer stock in case of delayed deliveries by the suppliers.

(f)Helps in minimizing the loss due to obsolescence, damages.

(g)Ensures smooth functioning of its various departments by maintaining reasonable stocks.

Basic Characteristics of an Inventory Control System:

(1) Relevant Inventory Costs: The costs that are affected by the firm's decision to maintain a particular level of stock are called relevant costs. It includes the following:

(a) Purchase Cost: It is the actual price paid for the procurement of items and includes direct material, direct labour and direct expenses.

(b) Ordering Costs/Acquisition/Replenishing or Set-up Costs: It is the cost of placing an order and it varies with individual organization. This cost is independent of the size of the order, but varies with the number of orders placed during a given period of time.

Ordering Cost = Cost per order * Number of orders placed during a period.

(c) **<u>Inventory Carrying/Holding Costs</u>:** Holding cost is associated with storing an item in inventory. It is proportional to the amount of inventory and the time over which it is held. It includes the following:

(i) The cost of capital tied up in the inventory.

(ii) Cost of warehouses, racks, salary of warehouse employees.

(iii) Insurance Cost.

(iv) Pilferage cost.

(v) Obsolescence cost.

(vi) Handling cost associated with the movement of stocks such as cost of labour, and other machinery used for this purpose.

(vii) Storage operation cost including cost of maintaining inventory records and security expenses.

Carrying Cost = (Cost of carrying one unit of an item in the inventory for a given length of time) * (Average number of units of an item carried in the inventory for a given length of time).

(d) <u>Shortage or Stock out Cost</u>: Stock out cost is the cost of sales lost due to shortage of stock or loss of goodwill due to delay in supply of finished goods.

Total Inventory Cost = Purchase Cost + Ordering Cost + Carrying Cost + Shortage

Cost.

(2) <u>Demand</u>: The demand pattern of an item may be either:

(i) <u>Deterministic demand</u>: In this, the quantities needed over subsequent periods of time are known with certainty and expressed over equal periods of times.

(ii) <u>Probabilistic demand</u>: This occurs when quantities needed over a certain period of time are not known with certainty, but their pattern can be expressed by a known probability distribution. It can be stationary or non-stationary.

(3) <u>Ordering Cycle</u>: It refers to the time period between two successive placement of orders, which can be determined in following ways:

(i) Continuous Review: In this system, a record of the inventory level is updated until a certain lower limit (called re-order point) is reached at which point a new order of fixed amount is placed. This is also known as two-bin system.

(ii) Periodic Review: In this case, the orders are placed at equal intervals of time through periodic review. Quantity of orders is always decided on the basis of consumption between two review periods.

(4) Time Horizon: It is known as planning period over which inventory is to be controlled and is generally done on annual basis.

(5) Delivery Lag or Lead Time: It is the time between the requisition for an item and its receipt. It can be deterministic or probabilistic. It includes:

- **(i)** Administrative lead time.
- **(ii)** Supplier's lead time.
- **(iii)** Transportation lead time.
- **(iv)** Inspection lead time.

(6) Safety Stock: It is the minimum additional inventory which serve as a safety margin or cushion to meet an unanticipated increase in demand.

Safety stock = (Max. lead time – Avg. lead time) * Demand rate.

(7) Re-order Level: It is the amount of stock that is on hand at the time of the placement of the replenishment order. This level is enough to serve the customers during the lead time.

Re-order level = Safety stock + Lead time demand.

Steps involved in Inventory Model Building:

(a)Collect the data regarding the pattern of demand, the replenishment policy, planning horizon and relevant inventory costs.

(b)Develop the total annual inventory relevant cost.

(c)Transform the total annual cost from formula to a mathematical formula.

(d)Find the optimum quantity to be ordered and when to re-order and the total relevant cost.

(e)Estimate lead time, safety stock and reorder level.

(f)Develop the inventory model.

(g)Review the position and make suitable changes depending upon the current constraints.

The Economic Order Quantity (EOQ):

Economic order quantity is the size of the order representing standard quality of material and it is the one for which the aggregate of the costs of procuring the inventory and costs of holding the inventory is minimum.

Single-item deterministic inventory control models without shortages:

(a)The total number of units required for 1 year is known exactly.

(b)The demand is known & constant & is resupplied instantaneously.

(c)Orders are received instantaneously.

(d)Ordering costs are the same regardless of order size.

(e)The purchase price does not fluctuate during the period considered.

(f)There is sufficient space, and money to allow the procurement of any quantity desired.

Model 1: <u>Basic Economic Order Quantity Model with Infinite supply</u>:

<u>Basic Assumptions</u>:

(a) Annual demand, carrying cost and ordering cost is known with certainty and is constant over time.

(b) Avg. inventory level for a material is order quantity divided by two. It means that materials are entirely used up when the next order arrives.

(c) Volume discounts do not exist.

(d) There are not stockout costs, i.e. inventory is replenished immediately as the stock level is almost zero.

(e) Lead time is known with certainty.

(f) Infinite planning horizon is assumed.

Definitions:

(a)D = annual demand for a material (units per year)

(b)Q = Quantity of material ordered at each order point.

(c)C_h= Cost of carrying one unit in inventory for one year.

(d)C_o= Average cost of completing an order for material.

Formulas:

(a) Annual carrying cost = Avg. inventory level * carrying cost per unit per year.

= $(Q/2)* C_h$

(b) Annual Ordering cost = No. of orders placed per year * ordering cost per order.

= (annual demand/no. of units in each order) * ordering cost per order.

(c) Total annual cost = Annual carrying cost + Annual ordering cost.

(d) Average Inventory level = (Maximum inventory + Minimum inventory)/2

(e) Optimal time between two orders = EOQ/Annual demand

Q1: The XYZ manufacturing company has determined from an analysis of its accounting and production data for ‘part number alpha’, that its cost to purchase is Rs. 36 per order and Rs.2 per part. Its inventory carrying charge is 9% of the average inventory. The demand of this part is 10,000 units per annum. Determine:

(a) What should be the economic order quantity?

(b) What is the optimal number of orders?

Q2: A company, one of the A-class items, place 6 orders each of size 200 in a year. Given ordering cost = Rs. 600, holding cost = 40%, cost per unit = Rs. 40. Find out the loss to the company in not operating scientific inventory policy? What are your recommendations for the future?

Q3: The production department of a company works 50 weeks a year and has demand for an item which is constant at 100 units a week. The cost of each unit is Rs. 200 and the company aims for a return of 20% on capital invested. Annual warehouse costs are estimated to be 5% of the value of goods stored. The purchasing department of the company costs Rs. 4,50,000 a year and send out an average of 2,000 orders. Determine.

(a) Optimal order quantity for the item and optimal number of orders per year.

(b) Optimal time between two consecutive orders, and

(c) the minimum annual cost of carrying the item.

Model 2: Economic Production Quantity Model with Finite Replenishment (Supply) Rate: This model is similar to the previous one except that here supply is gradual rather than instantaneous.

There can be three possible relationships between demand rate & supply rate (or production rate):

(a)Demand rate > Production rate: In this case shortage will occur which is ignored in this model. It is assumed that shortage is not permitted.

(b)Demand rate = Production rate: In this case there will be no need of holding inventory in stock as both demand & production rates are equal.

(c)Demand rate < Production rate: In this case if production process remains continuous, inventory stock will go on increasing which is the main concern of this model.

- The problem involved in this model is to determine the optimal number of units to be produced in one production run as production continuity is not possible.
- This is useful where products must be ordered from a production department within the organisation.
- Production occurs at a specific rate greater than demand and finished goods are transferred gradually from production to finished goods inventory as they are produced.

Assumptions:

(a)Annual demand, carrying cost & ordering cost is known with certainty.

(b)No safety stock is utilized, materials are supplied and used at a uniform rate and materials are entirely used up when the next orders begins to arrive.

(c)Volume discounts do not exist.

(d)There are not stockout costs.

(e)Supply rate is greater than demand rate.

(f)Production begins immediately after production set-up.

Definitions:

(a) d = rate at which units are used out of inventory (D/number of working days)

(b) p = rate at which units are supplied to inventory.

Formulae:

(a) **Inventory Accumulation rate =** (p – d)

(b) **Maximum Inventory Level =** Inventory accumulation rate + delivery period

$$= \frac{(p-d)Q}{p}$$

(c) **Minimum inventory Level** = 0

(d) **Average Inventory Level =** $\frac{(p-d)Q}{2p}$

(f) Annual Carrying Cost = $\frac{(p-d)Q}{2p} * C_h$

(g) Annual Ordering Cost = $(D/Q) * C_o$

(h) Total annual cost = Annual carrying cost + Annual ordering cost.

(i) EOQ = $\sqrt{\frac{2\,DC_o\,(p)}{C_h\,(p-d)}}$

Q4: XYZ co. uses 10,000 units of a particular valve per year. Each valve costs Rs. 32. The production engineering department estimates set-up cost as Rs. 55 and the accounting department estimates the holding cost as 12.5% of the value of inventory. Replenishment rate is uniform 120 valves per day. Assuming 250 working days, calculate,

(a) Optimal order quantity

(b) Total inventory cost on the basis of optimal policy, and

(c) Optimal number of set-ups.

Q5: (i) At present a company is purchasing an item 'X' from outside suppliers. The consumption is 10,000 units per year. The cost of the item is Rs. 50 per unit and the ordering cost is estimated to be Rs. 1,000 per order. The cost of carrying inventory is 25%. If the consumption rate is uniform, determine the economic purchasing quantity.

(ii) In the above problem assume that company is going to manufacture the above item with the equipment that is estimated to produce 100 units per day. The cost of the unit thus produced is Rs. 35 per unit. The set-up cost is Rs. 1500 per set-up and the inventory carrying charge is 25%. How has your answer changed? Assume 250 working days in a year.

Model 3: <u>Inventory Control Model with Planned Shortages</u>:

•Earlier models were based on the assumption that shortage cost was not allowed. But in some situations economic advantage may be gained by allowing shortages to occur.

•One advantage is allowing shortages is to increase the cycle time so that the ordering costs is spread over a longer period.

•Another advantage is when the unit value of the inventory as well as carrying cost is high.

Assumptions:

(a)This model is also known as back order or planned shortages inventory model as Stockouts and back ordering is allowed.

(b)Back order is the situation in which a customer places an order, finds that the material is out of stock, and waits for the next shipment to arrive.

(c)This model assumes that the customer's sale will not be lost due to stockout.

Notations:

(a)C_b = Stockout or backorder cost per unit back-ordered per period.

(b)S = remaining units after the back-order is satisfied.

(c)Q – S = Number of shortages per order (back-order quantity)

(d)t_1 = Time during which inventory is on hand.

(e)T_2= Time during which a shortage exists.

(f)T = Time between receipt of orders.

Formulae:

$$\text{total shortage costs} = C_s \frac{S^2}{2Q}$$

$$\text{total carrying cost} = C_c \frac{(Q-S)^2}{2Q}$$

$$\text{total ordering cost} = C_o \frac{D}{Q}$$

$$TC = C_s \frac{S^2}{2Q} + C_c \frac{(Q-S)^2}{2Q} + C_o \frac{D}{Q}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}\left(\frac{C_s + C_c}{C_s}\right)}$$

$$S_{opt} = Q_{opt}\left(\frac{C_c}{C_c + C_s}\right)$$

Maximum number of back orders = Q (C_h / C_h + C_s)

Number of orders per year = D/Q

Q6: The demand for an item is deterministic and constant over the time and it is equal to 600 units per year. The per unit cost of the item is Rs. 50 while the cost of placing an order is Rs. 5. The inventory carrying cost is 20% of the cost of inventory per annum and the cost of shortage is Re. 1 per unit per month. Find the optimal ordering quantity when stockouts are permitted. If the stockouts are not permitted, what would be the loss to the company?

Q7: A dealer supplies you the following information with regard to a product dealt by him:

Annual demand:10,000 units; **Ordering cost:** Rs. 10/- order;

Price=Rs. 20/- unit; Inventory carrying cost: 20% of the value of inventory/- year.

The dealer is considering the possibility of allowing some back order to occur. He has estimated that the annual cost of backordering will be 25% of the value of inventory.

(a) What should be the optimal number of units of product he should buy in one lot?

(b) What quantity of the product should be allowed to be back-ordered, if any?

(c) What would be the maximum quantity of inventory at any time of the year?

(d) Would you recommend to allow back-ordering? If so, what would be the annual cost saving by adopting the policy of back-ordering.

Transportation Problem

- A transportation problem has a number of origins and a number of destinations. The transportation problem indicates the amount of consignment to be transported from various origins to different destinations so that the total transportation cost is minimized without disturbing the availability constraints and the requirement constraints.

- It is a special type of LPP where the objective is to minimise the cost of distributing a product to a number of destinations.

- In a transportation problem, the total amount of supply available at the origin and the total quantity demanded by the destinations are given in the problem. The cost of shipping a unit of goods from a known origin to a known destination is also given.

Types of Transportation Problem:

(a) **Balanced Transportation Problem:** When the total availability at the origin is equal to the total requirments at the destinations, it is called a balanced transportation problem.

(b) **Unbalanced Transportation Problem:** When the total availability at the origins is not equal to the total requirements at the destinations, it is called unbalanced transportation problem.

Basic Concepts:

(a) Feasible Solution: A feasible solution is that solution which satisfies the requirements of demand and supply. This is the solution which simultaneously removes all the existing surpluses and satisfies all the existing deficiencies.

(b) Initial Basic Feasible Solution: An initial basic feasible solution with an allocation of (m+n-1) number of variables, is called a basic feasible solution, i.e. "1" less than the number of rows and columns in the transportation table.

(c) Optimal Solution: A feasible solution is said to be optimal if it minimizes the total transportation cost.

(d) When the total capacity equals total requirement, the problem is called as balanced transportation problem, otherwise it is known as unbalanced transportation problem.

(e) Degenerate Problem: When the number of positive allocations at any stage of the feasible solution is less than the required number (n+m–1), the solution is said to be degenerate solution, otherwise non-degenerate solution.

Steps in solving transportation problem:

(a) First check whether total demand is equal to total supply. If yes, it is a balanced problem. If not, introduce a imaginary origin/destinations, to make the problem balanced one.

(b) Find an initial basic feasible solution.

(c) After obtaining the IBFS, check whether number of allocations are equal to (m+n-1); if yes proceed for optimal solution otherwise treat it as degeneracy.

(d) Calculate the total minimum cost.

Method of Finding an Initial Basic Feasible Solution:

(a) North – West Corner Method.

(b) Lowest Cost Entry Method.

(c) Vogel's Approximation Method (VAM)

North-West Corner Method:

(a) Firstly select the upper left hand corner cell which is in the North-West corner of the table, and allocate units equal to the minimum of S1 and D1 against the supply and demand quantities in the respective rows and columns.

(b) If the quantity of supply for the first row is exhausted then proceed down to the cell in the second row and first column and adjust as required.

(c) If the quantity of demand for the first column is exhausted then move horizontally to the next cell in the second column and second row and adjust as required.

(d) If, at any cell, the supply-quantity equals the demand-quantity then the next allocation has to be made in the cell either in the next row or column.

(e) The procedure can be continued until the available supply and demand quantities are allocated properly to the concerned cells as required.

Q: Solve the following transportation problem by NWC method:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Requirement (demand)	**5**	**8**	**7**	**14**	

Least Cost Method (Matrix Minima Method): According to this method, we take into consideration the lowest cost for the purpose of appropriate allocation. The following steps are being followed:

(a) Select the cell with the lowest cost amongst all the figures of cost in all the rows and columns of the given data.

(b) To this selected cell in (a) allocate all the possible number of units either supply or demand as the case may be.

(c) According to step (b) when the demand gets satisfied or the supply gets exhausted, we eliminate the concerned row or column. Steps (a) and (b) is repeated till the supply at various plants gets exhausted according to the demand from the concerned warehouses.

Steps in solving transportation problem:

(a) First check whether total demand is equal to total supply. If yes, it is a balanced problem. If not, introduce a imaginary origin/destinations, to make the problem balanced one.

(b) Find an initial basic feasible solution.

(c) After obtaining the IBFS, check whether number of allocations are equal to (m+n-1); if yes proceed for optimal solution otherwise treat it as degeneracy.

(d) Calculate the total minimum cost.

Q: Solve the following transportation problem with the help of least cost method.

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Requirement (demand)	**5**	**8**	**7**	**14**	

VOGEL’S APPROXIMATION METHOD (VAM):

(a) For each row and column, find the difference between the smallest cost and the next smallest cost in the concerned row or column. Each such difference is called a penalty.

(b) Find the row or column with the largest penalty and in this row or column select the cell having the smallest cost and allocate the maximum possible quantity to this cell. The row or column for which the supply gets exhausted or the demand gets satisfied, becomes a deleted row or column.

(c) The process stated in (a) and (b) is repeated till the entire supply at the different plants gets exhausted by satisfying the demand at the various wareshouses.

Q: Solve the following transportation problem with the help of Vogels' Approximation method:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Requirement (demand)	**5**	**8**	**7**	**14**	

Methods of finding the Optimum Solution:

- Find an initial basic feasible solution using any of the three methods.
- Test the initial basic feasible solution for optimality using any of the following methods:

(a) Stepping Stone Method.

(b) Modified Distribution method (MODI)

STEPPING STONE METHOD:

(a) Find the initial basic feasible solution by using any of the methods.

(b) Identify each unoccupied cell and follow its closed path, so as to determine its net cost change. If all the net cost changes have zero or positive sign, then the solution becomes optimal. If any negative net change or changes is found, then find out the unoccupied cell with the largest negative value and carry out allocation process.

(c) After finding the quantity to be appropriately allocated to the selected unoccupied cell, follow the closed path for this cell and with reference to the negative sign in this path identify the minimum quantity. By allocating this quantity find the new solution.

Q: Solve the following transportation problem and find the optimal solution:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	30	25	40	20	**100**
F2	29	26	35	40	**250**
F3	31	33	37	30	**150**
Requirement (demand)	**90**	**160**	**200**	**50**	

MODIFIED DISTRIBUTION METHOD (MODI):

In the MODI method, the improvement cost of all the unoccupied cells are calculated, without drawing their respective closed paths. Only one closed path is drawn after the unused square with the highest negative value is identified.

Various steps of the MODI method are:

(a) Determine the IBFS using any of the three methods.

(b) For each occupied cell in the current solution, solve the system of (m+n-1) equations:

$$U_i + V_j = C_{ij}$$

Since the number of unknown **Ui** and **Vj** are (m + n), we can assign an arbitrary value to **Vj = 0** or **Ui = 0** for each unoccupied basic cell and enter them in the upper right corner of the corresponding cell.

$$\Delta ij = \mathbf{Cij} - \mathbf{(Ui + Vj)}.$$

(c) Examine the sign of each opportunity cost. If $\Delta ij > 0$, the given solution is an optimal one. If atleast one $\Delta ij < 0$, the given basic feasible solution is not an optimum one and further savings in the transportation cost are possible.

(d) Select the unoccupied cell with the largest negative Δij as the cell to be included in the next solution.

(e) Trace a closed path for the unoccupied cell selected in above step.

(f) Assign alternate plus and minus sign.

(g) Determine the maximum number of units that should be shipped to this unoccupied cell.

(h) This whole procedure is repeated till we obtain an optimal solution.

Q: Find the optimal solution of the following transportation problem using VAM method:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	12	**7**
F2	70	30	40	60	**10**
F3	40	10	60	20	**18**
Requirement (demand)	**5**	**8**	**7**	**15**	

Q: Solve the following transportation problem and find the optimal solution using Vogel's Approximation method:

Factory	WAREHOUSES					Capacity (Supply)
	W1	W2	W3	W4	W5	
F1	4	1	3	4	4	**60**
F2	2	3	2	2	3	**35**
F3	3	5	2	4	4	**40**
Requirement (demand)	**22**	**45**	**20**	**18**	**30**	

Variations in Transportation Problem:

(a) **Unbalanced transportation problem:** The transportation problem in which the total availability is not equal to the total requirement is called unbalanced transportation problem.

STEPS:

(b) Add one more imaginary origin or warehouse where availability is more as compared to requirement.

(c) Add one more imaginary destination where requirement is more as compared to availability. The main condition of adding a new destination is that the true transportation cost of the units should be kept zero.

Q: Consider the following transportation problem and find out the optimum answer.

Factory	WAREHOUSES			Capacity (Supply)
	W1	W2	W3	
F1	8	16	16	**152**
F2	32	48	32	**164**
F3	16	32	48	**154**
Requirement (demand)	**144**	**204**	**82**	

(b) **Degeneracy:** If the number of occupied cells is less than (m + n – 1) at any stage of the solution, then the problem is said to have a degenerate solution. Degeneracy can occur at 2 stages: (1) at the initial solution (2) During testing of optimal solution.

(i) **Degeneracy occurs at the initial solution:** To resolve degeneracy at the initial stage, an artificial quantity denoted by the greek letter (€) is used in one or more of the unoccupied cells so that the number of occupied cells is (m + n – 1)

The quantity of € is so small that it does not affect the supply and demand constraints.

The € value is assumed to be zero when actually used in the movement of the goods from one cell to another.

Once an "€" is introduced into the solution, it will remain there until degeneracy is removed or a final solution is arrived at.

Q: Determine an optimal distribution for the company in order to minimize the total cost.

Factory	WAREHOUSES					Capacity (Supply)
	W1	W2	W3	W4	W5	
F1	5	8	6	6	3	**80**
F2	4	7	7	6	6	**50**
F3	8	4	6	6	3	**90**
Requirement (demand)	**40**	**40**	**50**	**40**	**80**	

Q: Solve the following transportation problem:

Factory	WAREHOUSES			Capacity (Supply)
	W1	W2	W3	
F1	50	30	220	**1**
F2	90	45	170	**3**
F3	250	200	50	**4**
Requirement (demand)	**4**	**2**	**2**	

(b) <u>Degeneracy occurs during the testing of the Optimal Solution:</u>

Degeneracy during the solution stage occurs when the inclusion of the unoccupied cell with maximum negative opportunity costs results in vacating of 2 or more occupied cells. This problem is resolved by allocating € to one or more of the vacated cells to complete the required (m + n – 1) conditions.

Q: Solve the transportation Problem:

	W	X	Y	SUPPLY
A	7	3	6	5
B	4	6	8	10
C	5	8	4	7
D	8	4	3	3
DEMAND	5	8	10	

Transportation problems of Maximum Profit:

In some transportation problems, information is given in the form of per unit profit, to solve such problems highest profit value from the profit matrix is selected and all values are subtracted from this highest value. After obtaining this matrix, same procedure is applied.

Q: Solve the following transportation problem for maximum profit:

	A	B	C	D	SUPPLY
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300
DEMAND	180	320	100	400	

REPLACEME NT THEORY

- Replacement theory is concerned with the problem of replacement of machines due to their deteriorating efficiency, failure or breakdown.
- **Replacement theory deals with:**

(a) When existing items have outlived their effective lives and it may not be economical to continue with them.

(b) Items which might have been destroyed either by accident or otherwise.

(c) Replacement of items that deteriorate with time.

(d) Replacement of an equipment that becomes out of date due to new developments.

(e) Improved technology has given access to much better and technically superior products.

Causes of Replacement:

(a)Deterioration: It involves reduction in the value of an asset due to wear & tear.

(b)Obsolescence: It may occur due to advancement of technology.

(c)Inadequacy: The capacity of an equipment may be inadequate to meet the demand or to increase the production to a desired level.

(d)Working conditions: Old equipment may become noisy, smoking or unsafe for the workers.

(e)Economy: Existing equipment may outlived their effective life and it is not viable to continue with them.

(f)Sudden failure: The existing equipment may destroy all of a sudden.

Characteristics of Replacement:

(a)Replacement reduces maintenance cost but it involves a high average capital cost.

(b)Equipment must be constantly reviewed and updated at certain intervals to avoid its obsolescence rather than watching until it is physically out of order.

Replacement models and their solutions: It involves comparison of alternative replacement policies and the factors relevant to the replacement are as follows:

(a)Technical: It includes deterioration, obsolescence and inadequacy.

(b)Financial: It includes initial cost, running cost, labour cost, operating cost, salvage value and insurance.

Various types of Replacement Problems:

(a)Replacement policy for equipment that deteriorates gradually.

(b)Replacement policy for items that fail suddenly.

(c)Staff replacement problems.

Failure Mechanism of Items: There are two types of failure:

(a) Gradual Failure: The failure mechanism under gradual failure is progressive, i.e. as the life of an item increases, its efficiency deteriorates resulting in:

(i) increased expenditure for operating costs.

(ii) decreased productivity of the equipment.

(iii) decrease in the value of equipment, i.e. resale value decreases.

(b) Sudden Failure: This class of failure is applicable to those items that do not deteriorate with service but which ultimately fail after a period of use. Sudden failures can be progressive, retrogressive or random.

(i) Progressive Failure: Under this, the probability of failure increases with increase in the life of an item.

(ii) Retrogressive Failure: Under this, items have more probability of failure in the beginning of their life and as time passes, the chances of failure become less.

(iii) Random Failure: Under this, constant probability of failure is associated with items that fail from random causes such as physical shocks, not related to age.

Methodology of solving replacement problems:

(a)Identify the items to be replaced and also their failure mechanism which can be sudden or gradual.

(b) Collect the data relating to the depreciation cost and the maintenance cost over a time period for the items which follow gradual failure. In case of sudden failure, collect the data for failure rates and cost of preventive replacement.

(c) By using the above data, develop a suitable model in OT for determining the exact time of replacing the equipment.

Replacement Policy for Equipment/Asset ;which deteriorates gradually:

- It is economical to replace the equipment with a new one when operational efficiency of the equipment deteriorate by comparing the number of alternative choices available on the basis of average maintenance and operating cost involved.

(a) Replacement policy for items whose running cost increases with time without change in the value of money during a period: Cost of an equipment over a given period of time has the following three elements:

(i) C = Purchase price of the equipment.

(ii) S = Scrap value of the equipment taken to be same over n years.

(iii) $g(t)$ = Maintenance/running cost of the equipment at time t.

Thus, **the annual cost of the machine at any time t**

= Capital cost – scrap value + maintenance (or running) cost at time t.

<u>Now g(t) can be discrete or continuous function of time.</u>

- If g(t) is a discrete function, then summation sign is used to find the total maintenance cost in a given period.

$$g(t) = \sum_{t=0}^{n} g(t)$$

- If g(t) is a continuous function, then integrals can be used.

$$g(t) = \int_0^n g(t)dt$$

Average Cost, A(n) would be defined as:

$$\mathrm{A(n)} = \frac{1}{n}\left[c - s + \sum g(t)\right]$$

- It is advisable to replace the item when A(n) is maximum. However, A(n) will be minimum when:

  $g(n) < A(n-1)$ and

  $A(n) < g(n+1)$

- Replace the item in the year in which the average cost is minimum.

Q1: A firm is using a machine whose purchase price is Rs. 13,000. The installation charges amount to Rs. 3600 and the machine has a scrap value of only 1600. The maintenance cost in various years is given in the following table. The firm wants to determine after how many years should the machine be replaced on economic considerations, assuming that the machine replacement can be done only at the year ends.

Year	1	2	3	4	5	6	7	8	9
Cost (Rs)	250	750	1000	1500	2100	2900	4000	4800	6000

Q2: A company has a machine whose purchase price is Rs. 80,000. The expected maintenance costs and resale price in different years are as given below. After what time interval, in your opinion, should the machine be replaced?

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Maintenance cost	1000	1200	1600	2400	3000	3900	5000
Resale value ('000)	75	72	70	65	58	50	45

Q3: (a) Machine A costs Rs. 9,000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2,000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine?

(b) Machine B costs Rs. 10,000. Annual operating costs are Rs. 400 for the first year, and then increase by Rs. 800 every year. You now have a machine of type A which is one year old. Should you replace it with B; if so when?

Replacement Policy of Equipment/Asset whose running cost increases with time but value of money changes at constant rate:

- In this case, we have to discount the future payments on maintenance cost so as to express them all in terms of present value. In this, a machine has to be replaced when weighted average annual cost is minimum.

- **Notations:**

  (a) C = initial cost (or purchase price) of the item to be replaced.

  (b) g(t) = Running (or maintenance) cost in *t*th year.

  (c) r = rate of interest

  (d) v = 1/(1+r) is the present worth of a rupee to be spend a year hence.

Steps to be followed:

(a)In first column, write maintenance costs of machine for different years.

(b)In second column, write discounting factor indicating the present value.

(c)In third column, calculate discounted running cost by multiplying column (a) and (b).

(d)In fourth column, calculate cumulative discounted running cost.

(e)Then, calculate cumulative discounted factor shown in column (b).

(f)Finally, in the last column calculate weighted average annual cost by dividing the cumulative discounted running cost (column d) by cumulative discounted factor (column e).

(g)Replace the machine in the year in which the weighted average annual cost is minimum.

Q4: The initial cost of a machine is Rs. 30,000 and running or operating expenditure which increases with age of the machine is given below. What is the replacement policy? When this machine should be replaced? It is given that the rate of interest is 10% and scrap value is nil.

Year	1	2	3	4	5	6	7
Running Cost (Rs.)	5,000	6,000	8,000	10,000	13,000	16,000	20,000

Q5: The initial cost of an item is Rs. 15,000 and maintenance or running costs for different years are given below. What is the replacement policy to be adopted if the capital is worth 10% and there is no salvage value?

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Running Cost (Rs.)	2500	3000	4000	5000	6500	8000	10000

Q6: An engineering company is offered two types of material handling equipments A and B. A is priced at Rs. 60,000 including cost of installation and the costs for the operation and maintenance are estimated to be Rs. 10,000 for each of the first five years, increasing every year by Rs. 3,000 per year in the sixth and subsequent years.

Equipment B, with rated capacity same as A, requires an initial investment of Rs. 30,000 but in terms of operation and maintenance costs more than A. These costs for B are estimated to be Rs. 13,000 per year for the first six years, increasing every year by Rs. 4,000 per year from seventh year onwards. The company expects a return of 10% on all its investments. Determine which equipment the company should buy.

Replacement of items that fail completely:

•There are situations where the failure of a certain item occurs all of a sudden instead of gradual deterioration e.g. electric light bulb, which result in complete breakdown of a system.

•This breakdown implies loss of production and idle inventory labor.

•It is very difficult to determine the probability of failure of any item in the system and it can be done by assigning the probability distribution of failures.

•It is assumed that failures occur only at the end of the period.

•Thus, the main objective is to find the time period ‘t’ which minimizes the total cost involved for the replacement.

There are two types of replacement policies:

(a)Individual Replacement Policy: Under this policy an item is replaced immediately whenever it fails.

(b)Group Replacement Policy: Under this policy, decision is taken as to when all the items must be replaced irrespective of the fact that items have failed or not, with the provision that if any item fails before the optimal time, it may be replaced individually. It requires twofold considerations:

(i) the rate of individual replacement during the period, and

(ii) the total cost incurred for individual and group replacements during the selected interval.

- The period for which the total cost is minimum will be the optimal period for replacement.

- For group replacement policy one should know the probability of failure, loss incurred due to these failures, cost of individual replacements and costs of group replacements.

- Replace the group of items at the end of the period if the cost of individual replacement for that period is greater than the average cost per period.

Q: The management is considering the periodic replacement of light bulbs fitted in its rooms. There are 500 rooms in the hotel and each room has 6 bulbs. The management is now following the policy of replacing the bulbs as they fail at a total of Rs. 30 per bulb. The management feels that this cost can be reduced to Rs. 10 by adopting replacement method. On the basis of the information given below, evaluate the alternative and make a recommendation to the management:

Months of use	**1**	**2**	**3**	**4**	**5**
% of bulbs failing by that month	10	25	50	80	100

Q: The following failure rates have been observed for a certain type of transistors in a digital computer. The cost of replacing an individual failed transistor is Rs. 1.25. The decision is made to replace all these transistors simultaneously at fixed intervals, and to replace the individual transistors as they fail in service. If the cost of group replacement is 30 paise per transistor, what is the best interval between group replacement? At what group replacement price per transistor would a policy of strictly individual replacement become preferable to the adopted policy?

End of the week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Prob. of failure to date	0.05	.13	.25	.43	.68	.88	.96	1.00